ELECTRIC FIELD FROM A POINT SOURCE IN THE ZONE OF A WELL WITH CONSIDERATION OF THE STRAINED STATE OF THE BED

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We will investigate an electric field from a point source in an unbounded space having a cylindrical channel with consideration of inhomogeneous strain of the medium. An analogous problem but without consideration of the strained state of the medium was investigated in [1, 2] with reference to problems of electrical prospecting of wells. The need to take into account the strained state of the medium is related with the fact that experimental studies [3-5] indicate a considerable dependence of the electrical conductivity of a number of materials of practical interest on their strained state.

1. Since the strained state of a material is determined by the strain tensor ε_{ij}, the electrical conductivity of the material should be some function of this tensor. This dependence should be invariant relative to the choice of the coordinate system, i.e., the conductivity should also be some tensor k_{ij} and be associated by a functional relation with the strain tensor ε_{ij}.

The most general form of such a functional tensor relation is [6]

$$k_{ij} = F(A_1, A_2, A_3)\,\delta_{ij} + \Phi(A_1, A_2, A_3)\,\varepsilon_{ij} + W(A_1, A_2, A_3)\,\varepsilon_{ij}^2 \tag{1.1}$$

where F, Φ, W are arbitrary functions; A_1, A_2, A_3 are invariants of tensor ε_{ij}; δ_{ij} is a unit tensor. Considering the strains ε_{ij} to be small, we can linearize functional relation (1.1). After expanding (1.1) in series and limiting ourselves to term of the first order of smallness, we obtain

$$k_{ij} = (\alpha + \delta\theta)\,\delta_{ij} + \gamma\varepsilon_{ij} \tag{1.2}$$

where α, δ, γ are constant moduli; $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ is the first invariant of tensor ε_{ij}; α is the electrical conductivity in a state taken to be unstrained.

The coefficients δ, γ characterize the effect of the strain of the material on its conductivity. The coefficients δ, γ must be found experimentally, whereby two experiments on measuring the conductivity in uniform and uniaxial compression are sufficient for finding them.

Thus, on the basis of (1.2) the current density **i** in the strained medium is

$$\mathbf{i} = k_{ij}\nabla u \tag{1.3}$$

where u in the electric field potential.

The equation of a constant electric field in a deformable medium on the basis of (1.2) and (1.3) will be

$$\nabla k_{ij}\nabla u = 0 \tag{1.4}$$

2. With consideration of the dependence of the conductivity of the material on the strained state established above the problem has the following statement: it is required to find the electric field from a dc point source I in an unbounded space with a cylindrical channel of radius r_0, the source being located on the channel axis. The electrical conductivity of the medium depends on its strained state which arises due

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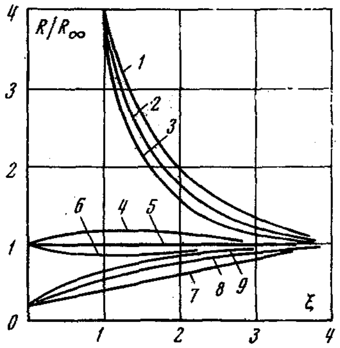

Fig. 1

to the pressure of uniform compression at infinity p_2 and pressure in the channel p_1. The channel is filled with fluid with electrical conductivity σ. This statement of the problem is possible if an electric probe is in a well filled with drilling fluid, whereby the thickness of the bed is much greater than the well radius and much less than the depth of occurrence of the bed. In this case the bed can be regarded as unbounded and under the effect of pressure p_2 at infinity equal to the rock pressure of the overlying soil layers. Pressure p_1 corresponds to the pressure of the drilling fluid in the region of the bed.

The problem is solved on the assumption that the material of the medium is linearly elastic and the stress tensor σ_{ij} is associated with the strain tensor ε_{ij} by the relation

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \tag{2.1}$$

Here λ, μ are the Lamé constants. The boundary conditions of the elastic problem have the following form in cylindrical coordinates:

$$\begin{array}{lll} \sigma_{rz}=0, & \sigma_{rr}=-p_1 & (r=r_0) \\ \sigma_{rz}=0, & \sigma_{rr}=-p_2 & (r\to\infty) \\ \sigma_{rz}=0, & \sigma_{zz}=-p_2 & (z\to\pm\infty) \end{array} \tag{2.2}$$

By virtue of axial symmetry and the boundary conditions there will exist in the medium three nontrivial components of the stress tensor σ_{rr}, σ_{zz}, $\sigma_{\varphi\varphi}$ and three nontrivial components of the strain tensor ε_{rr}, ε_{zz}, $\varepsilon_{\varphi\varphi}$. The components of the displacement vector in radial and axial directions u_r, u_z satisfying the equations of the axisymmetric elasticity theory and boundary conditions (2.2) have the form

$$u_r = -\frac{p_2}{3\lambda+2\mu}r - \frac{p_2-p_1}{2\mu}\frac{r_0^2}{r}, \qquad u_z = -\frac{p_2}{3\lambda+2\mu}z \tag{2.3}$$

The components of the strain tensor corresponding to the displacement vector (2.3) will be

$$\begin{gathered} \varepsilon_{rr} = -\frac{p_2}{3\lambda+2\mu} + \frac{p_2-p_1}{2\mu}\frac{r_0^2}{r^2} \\ \varepsilon_{\varphi\varphi} = -\frac{p_2}{3\lambda+2\mu} - \frac{p_2-p_1}{2\mu}\frac{r_0^2}{r^2} \\ \varepsilon_{zz} = -\frac{p_2}{3\lambda+2\mu}, \qquad \varepsilon_{rz} = \varepsilon_{r\varphi} = \varepsilon_{z\varphi} = 0 \end{gathered} \tag{2.4}$$

3. On the basis of (1.2), (2.4) the conductivity tensor for region $r > r_0$ is written

$$\begin{gathered} k_{rr} = \alpha - (3\delta+\gamma)\frac{p_2}{3\lambda+2\mu} + \gamma\frac{p_2-p_1}{2\mu}\frac{r_0^2}{r^2} \\ k_{\varphi\varphi} = \alpha - (3\delta+\gamma)\frac{p_2}{3\lambda+2\mu} - \gamma\frac{p_2-p_1}{2\mu}\frac{r_0^2}{r^2} \\ k_{zz} = \alpha - (3\delta+\gamma)\frac{p_2}{3\lambda+2\mu}, \qquad k_{rz} = k_{r\varphi} = k_{z\varphi} = 0 \end{gathered} \tag{3.1}$$

The equation for the distribution of the electric potential u_2 in region $r > r_0$ on the basis of (3.1), (1.4) after introducing the dimensionless parameters $\rho = r/r_0$, $\xi = z/r_0$ will take the form

$$\left(1-\frac{b}{\rho^2}\right)u_{2,\rho\rho} + \frac{u_{2,\rho}}{\rho}\left(1+\frac{b}{\rho^2}\right) + u_{2,\xi\xi} = 0 \tag{3.2}$$

Here b is a dimensionless parameter

$$b = -\frac{\gamma(3\lambda+2\mu)}{\alpha(3\lambda+2\mu)-(3\delta+\gamma)p_2}\frac{p_2-p_1}{2\mu} \tag{3.3}$$

In the absence of a pressure drop p_2-p_1, $b=0$ and the Laplace equation is obtained for the distribution of the electric field. On finding the particular solutions of Eq. (3.2) by the method of separation of variables the following expression is obtained:

$$u = [C_1 I_0(\gamma\sqrt{\rho^2-b}) + C_2 K_0(\gamma\sqrt{\rho^2-b})]\cos\gamma\xi \tag{3.4}$$

Here I_0, K_0 are Bessel functions of an imaginary argument of the first and second kind; C_1, C_2 are constants of integration. In the region $r < r_0$ filled with fluid with conductivity σ the potential u_1 satisfies the equation

$$u_{1,\rho\rho} + \frac{u_{1,\rho}}{\rho} + u_{1,\xi\xi} = 0 \tag{3.5}$$

It is required to find the electric potential satisfying Eq. (3.5) in region $r < r_0$ and Eq. (3.2) in region $r > r_0$, whereby at the boundary $\rho = 1$ the coupling conditions, which consist in equality of the potentials u_1, u_2 and equality of the normal components of the current through the cylindrical surface $\rho = 1$, should be fulfilled:

$$u_1 = u_2, \quad \sigma u_{1,\rho} = \left[\alpha - (3\delta + \gamma)\frac{p_2}{3\lambda + 2\mu} + \gamma\frac{p_2 - p_1}{2\mu}\right] u_{2,\rho} \tag{3.6}$$

The point source I is located at point $\rho = 0$, $\xi = 0$, and therefore the potential u_1 should have at this point a singularity corresponding to the singularity of the point source, which is located in an unbounded homogeneous space with conductivity σ, i.e., at point $\rho = 0$, $\xi = 0$ the potential u_1 should have the singularity [1, 7]

$$u_1 = \frac{I}{4\pi\sigma r_0 \sqrt{\rho^2 + \xi^2}}\left[1 + O\left(\frac{1}{\sqrt{\rho^2 + \xi^2}}\right)\right] \tag{3.7}$$

The potential should approach zero with distance from the source:

$$u_1 \to 0, \quad \xi \to \pm\infty; \qquad u_2 \to 0, \quad \sqrt{\rho^2 + \xi^2} \to \infty \tag{3.8}$$

4. The general solutions of Eqs. (3.2) and (3.5) with consideration of the boundary conditions and coupling conditions are selected in the form

$$u_1 = \int_0^\infty C(\gamma) K_0(\gamma\rho) \cos\gamma\xi \, d\gamma + \int_0^\infty A(\gamma) I_0(\gamma\rho) \cos\gamma\xi \, d\gamma \tag{4.1}$$

$$u_2 = \int_0^\infty B(\gamma) K_0(\gamma\sqrt{\rho^2 - b}) \cos\gamma\xi \, d\gamma \tag{4.2}$$

With the use of the known integral relation [8]

$$\int_0^\infty K_0(\gamma\rho) \cos\gamma\xi \, d\gamma = \frac{\pi}{2}\frac{1}{\sqrt{\rho^2 + \xi^2}}$$

we can fulfill the condition at the singular point $\rho = 0$, $\xi = 0$, having set

$$C(\gamma) = \frac{I}{2\pi^2 r_0 \sigma} \tag{4.3}$$

To determine coefficients $A(\gamma)$, $B(\gamma)$ it is sufficient to use the coupling conditions at the boundary $\rho = 1$, which, if we introduce notation, are written

$$\beta = \sqrt{1 - b}, \qquad s = \frac{\sigma(3\lambda + 2\mu)}{\alpha(3\lambda + 2\mu) - (3\delta + \gamma)p_2} \tag{4.4}$$

$$\int_0^\infty \left[\frac{I}{4\pi^2 r_0 \sigma} K_0(\gamma) + A(\gamma) I_0(\gamma) - B(\gamma) K_0(\gamma\beta)\right] \cos\gamma\xi \, d\gamma = 0 \tag{4.5}$$

$$\int_0^\infty \left[sA(\gamma) I_1(\gamma) - \frac{sI}{4\pi^2 r_0 \sigma} K_1(\gamma) + \beta B(\gamma) K_1(\gamma\beta)\right] \cos\gamma\xi \, d\gamma = 0 \tag{4.6}$$

Equating the integrands of (4.5), (4.6) to zero, we obtain the following expressions for the functions:

$$A(\gamma) = \frac{I}{2\pi^2 r_0 \sigma} \frac{sK_1(\gamma) K_0(\gamma\beta) - \beta K_0(\gamma) K_1(\gamma\beta)}{\beta I_0(\gamma) K_1(\gamma\beta) + sI_1(\gamma) K_0(\gamma\beta)} \tag{4.7}$$

$$B(\gamma) = \frac{sI}{2\pi^2 r_0 \sigma\beta} \frac{1}{\gamma[\beta I_0(\gamma) K_1(\gamma\beta) + sI_1(\gamma) K_0(\gamma\beta)]} \tag{4.8}$$

Thus the distribution of the potentials u_1, u_2 is given by the formulas

$$u_1 = \frac{I}{2\pi^2 r_0 \sigma}\left\{\int_0^\infty K_0(\gamma\rho) \cos\gamma\xi \, d\gamma + \int_0^\infty \frac{sK_1(\gamma) K_0(\gamma\beta) - \beta K_0(\gamma) K_1(\gamma\beta)}{\beta I_0(\gamma) K_1(\gamma\beta) + sI_1(\gamma) K_0(\gamma\beta)} I_0(\gamma\rho) \cos\gamma\xi \, d\gamma\right\} \tag{4.9}$$

$$u_2 = \frac{sI}{2\pi^2 r_0 \sigma\beta} \int_0^\infty \frac{K_0(\gamma\sqrt{\rho^2 - b}) \cos\gamma\xi \, d\gamma}{\gamma[\beta I_0(\gamma) K_1(\gamma\beta) + sI_1(\gamma) K_0(\gamma\beta)]} \tag{4.10}$$

We see from (4.9), (4.10) that the distribution of the potential to within the factor depends on two dimensionless parameters s, β, whereby when $\beta = 1$ a pressure drop $p_2 - p_1$ is absent and the solution known from the literature [1] is obtained.

The integrals of (4.9), (4.10) are convergent, since the integrands have a logarithmic singularity and do not have poles. Actually, if at some point γ_0 the integrand had a pole, the relation

$$\frac{s}{\beta} = -\frac{I_0(\gamma_0) K_1(\gamma_0\beta)}{I_1(\gamma_0) K_0(\gamma_0\beta)} \tag{4.11}$$

would be fulfilled.

But for any γ

$$I_0(\gamma) > 0, \; I_1(\gamma) > 0, \; K_0(\gamma) > 0, \; K_1(\gamma) > 0$$

By definition $s > 0$, $\beta > 0$, and (4.11) cannot be fulfilled.

If a point source of power I were in a homogeneous medium with electrical resistance R_∞

$$R_\infty = \frac{3\lambda + 2\mu}{\alpha(3\lambda + 2\mu) - (3\delta + \gamma) p_2} \tag{4.12}$$

its potential u_3 would be given by the formula

$$u_3 = \frac{I R_\infty}{4\pi r_0 \sqrt{\rho^2 + \xi^2}} \tag{4.13}$$

The ratio of the measured potential u_1 to the potential u_3 at this point is in the terminology of [1] the relative apparent resistance

$$\frac{u_1}{u_2} = \frac{R}{R_\infty} = \frac{2}{\pi} \left\{ \sqrt{\rho^2 + \xi^2} \left\{ \int_0^\infty K_0(\gamma\rho) \cos \gamma\xi \, d\gamma + \int_0^\infty \frac{sK_1(\gamma) K_0(\gamma\beta) - \beta K_0(\gamma) K_1(\gamma\beta)}{\beta I_0(\gamma) K_1(\gamma\beta) + sI_1(\gamma) K_0(\gamma\beta)} I_0(\gamma\rho) \cos \gamma\xi \, d\gamma \right\} \right. \tag{4.14}$$

The value of the ratio R/R_∞ on the channel axis $\rho = 0$ was calculated for various values of parameters s, β and for different ξ. Figure 1 shows the curves of the relative apparent resistance R/R_∞ for values of s, β equal to 0.1-1.3, 0.1-1.0, 0.1-0.7, 1-1.3, 1-1, 1-0.7, 5-0.7, 5-1, 5-1.3 (curves 1-9 respectively).

We see from the figure that for large distances from the source the apparent resistance measured along the well axis approaches the resistance of the medium at infinity regardless of the values of s, β.

For small distances from the source the apparent resistance approaches the resistance of the medium in the region $r < r_0$. For intermediate distances the apparent resistance is affected by the difference in the resistances of the two zones and by the inhomogeneity of the strained state of the zone when $r > r_0$. The strained state of the zone $r > r_0$ affects the distribution of the potential owing to a change of resistance at infinity from uniform compression according to Eq. (4.12) and owing to distortion of the configuration of the curves of the apparent resistance by virtue of the inhomogeneity of the strained state characterized by deviation of the parameter β from unity.

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